

TOP-QUARK PAIR PRODUCTION BEYOND NEXT-TO-LEADING ORDER

Andrea Ferroglio

Johannes Gutenberg Universität Mainz

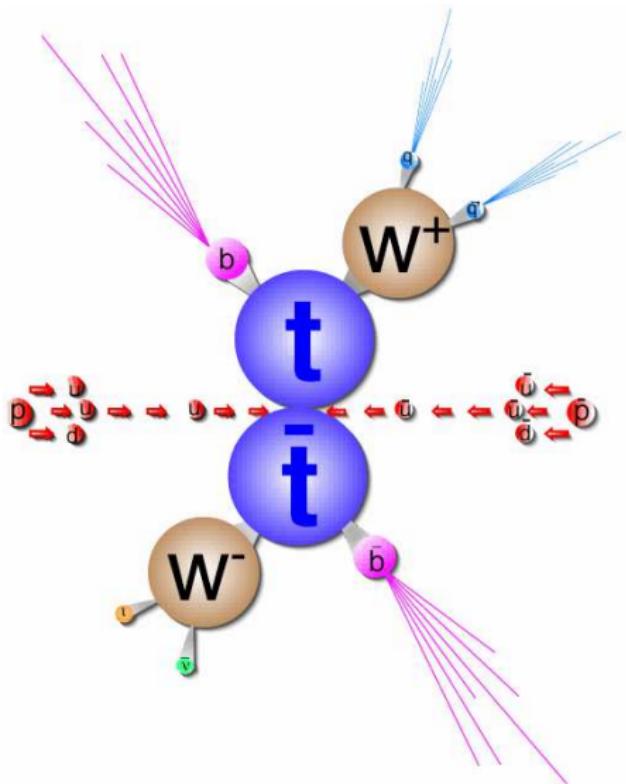
Stony Brook, June 23, 2010



OUTLINE

- 1 TOP-QUARK PAIR PRODUCTION AT HADRON COLLIDERS
- 2 INVARIANT MASS DISTRIBUTION
- 3 THE TOTAL CROSS SECTION

TOP-QUARK PAIR HADROPRODUCTION

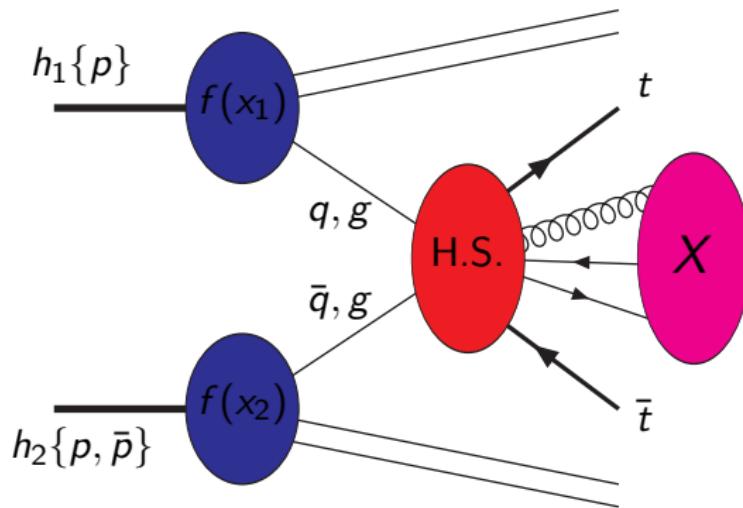


Several observables measured at Tevatron (few thousand observed events)

- Total Cross Section
 - Invariant Mass Distribution
 - Charge / Forward-Backward Asymmetry
 - ...
- Next 2 years at the LHC \sim few thousand observed events
- LHC ($\sqrt{s} = 14 \text{ TeV}$, $10 \text{ fb}^{-1}/\text{year}$)
⇒ millions of top-quarks / year

TOP QUARK PAIR HADROPRODUCTION & QCD

Top-quark pair production is a hard scattering process which can be computed in perturbative QCD

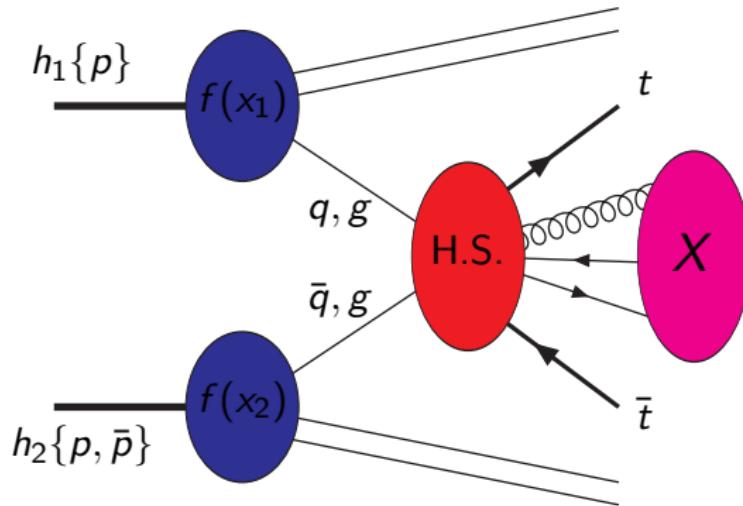


$$\sigma_{h_1, h_2}^{t\bar{t}} = \sum_{i,j} \int_0^1 dx_1 \int_0^1 dx_2 f_i^{h_1}(x_1, \mu_F) f_j^{h_2}(x_2, \mu_F) \hat{\sigma}_{ij}(s, m_t, \alpha_s(\mu_R), \mu_F, \mu_R)$$

$$s_{\text{had}} = (p_{h_1} + p_{h_2})^2, \quad s = x_1 x_2 s_{\text{had}}$$

TOP QUARK PAIR HADROPRODUCTION & QCD

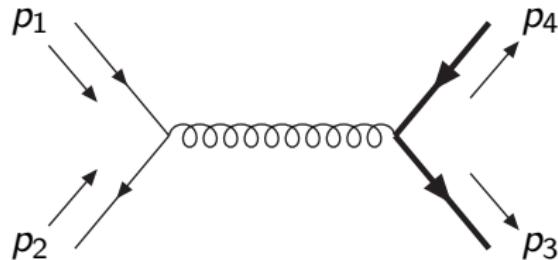
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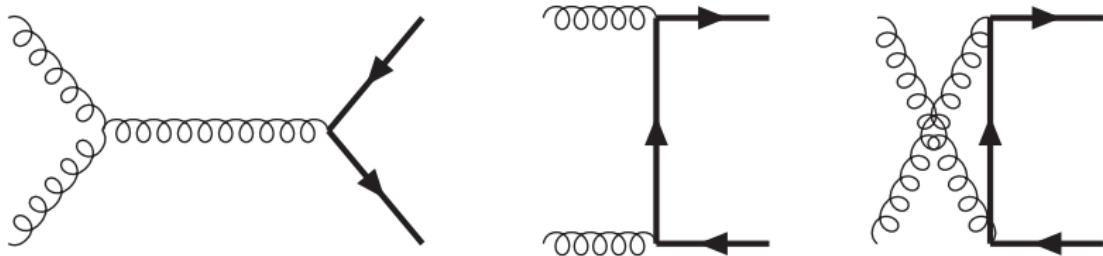
$$\sigma_{h_1, h_2}^{t\bar{t}}(s_{\text{had}}, m_t^2) = \sum_{ij} \int_{4m_t^2}^{s_{\text{had}}} ds \underbrace{L_{ij}(s, s_{\text{had}}, \mu_f^2)}_{\text{partonic luminosity}} \overbrace{\hat{\sigma}_{ij}(s, m_t^2, \mu_f^2, \mu_r^2)}^{\text{partonic cross section}}$$

TREE LEVEL QCD PARTONIC PROCESSES

$$q(p_1) + \bar{q}(p_2) \longrightarrow t(p_3) + \bar{t}(p_4)$$

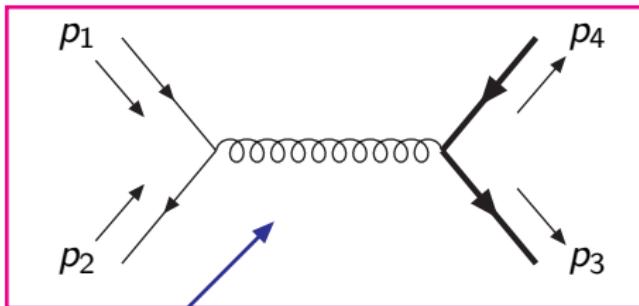


$$g(p_1) + g(p_2) \longrightarrow t(p_3) + \bar{t}(p_4)$$

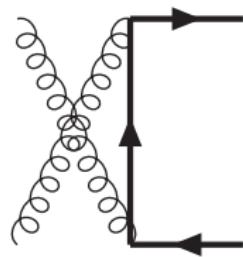
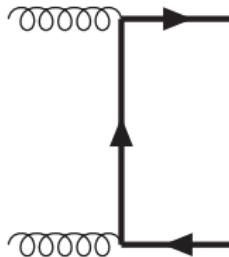
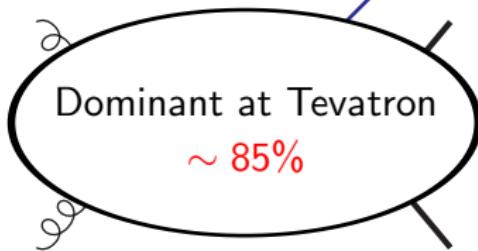


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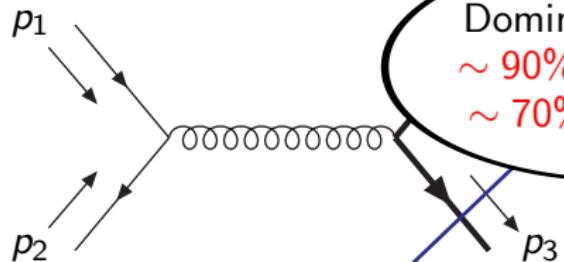


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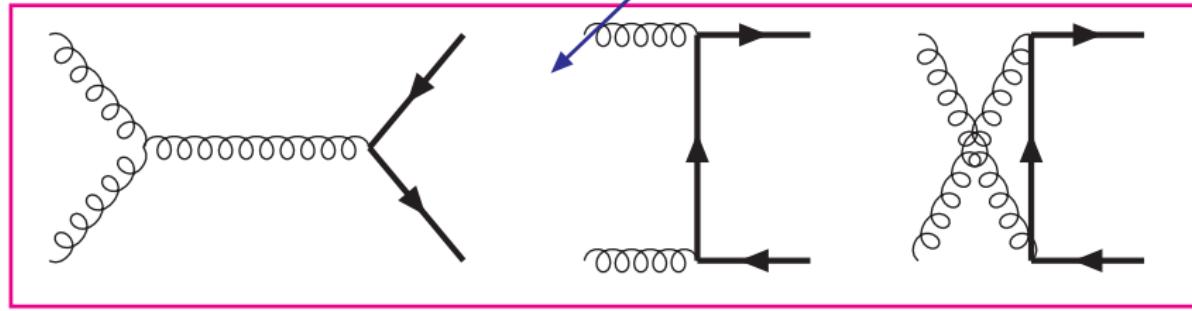


TREE LEVEL QCD PARTONIC PROCESSES

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NLO CORRECTIONS

The NLO corrections to top-quark pair production have been a subject of active research for more than 20 years
(too many authors to list them all here!)

- NLO QCD corrections to the total cross section
- NLO QCD corrections to the distributions (p_T , rapidity, invariant mass, ...)
- NLL resummation of threshold effects
- Mixed QCD-EW corrections
- NLO corrections keeping into account top spins and top decays
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At the LHC some observables (ex. total cross section) will be affected by experimental errors which are **smaller** than the current NLO + NLL theoretical uncertainties

To take full advantage of the LHC potential, we need to go
beyond the current NLO+NLL calculation

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COMPLETE NNLO

- Goal: to calculate all the virtual and real corrections
- Requires to use and develop cutting edge calculational techniques
- Very time consuming
- The only **ultimate solution** to the problem

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APPROXIMATE NNLO

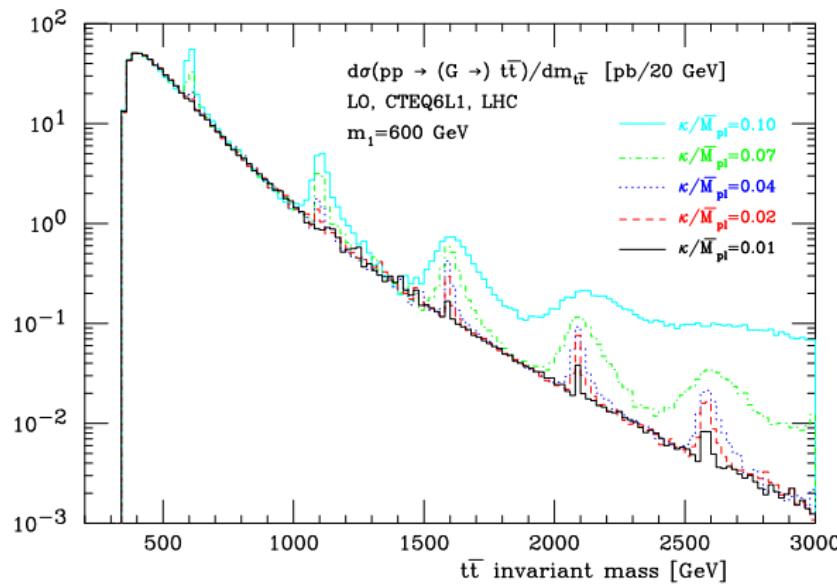
- Goal: to capture the numerically dominant NNLO corrections
- Can be done using Effective Field Theory methods
- One can obtain predictions relatively fast
- By construction, one introduces systematic uncertainties

**APPROXIMATE NNLO CALCULATIONS:
Partonic Threshold Expansion (and Resummation) for
the Invariant Mass Distribution**

APPROXIMATE NNLO FORMULAS FOR THE INVARIANT MASS DISTRIBUTION

Ahrens, AF, Neubert, Pecjak, and Yang ('09)

The distribution in the invariant mass $M^2 = (p_t + p_{\bar{t}})^2$ can be used to measure m_t , and to search for s -channel heavy resonances



Frederix and Maltoni ('07)

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$$\frac{d\sigma}{dM} = \frac{8\pi\beta}{3M} \int_{\tau}^1 \frac{dz}{z} \sum_{ij=(q\bar{q}, gg, \bar{q}q)} L_{ij} \left(\frac{\tau}{z}, \mu \right) C_{ij} (z, \dots, \mu)$$

We focus on the partonic threshold region $s \sim M^2$; $z \rightarrow 1$

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this is a virtual-soft approximation

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The distribution in the invariant mass $M^2 = (p_+ + p_-)^2$ can be used to measure

We can calculate the C_{ij} up to terms of $\mathcal{O}(1-z)$

Two quantities will be possible to predict accurately $d\sigma/dM$ if:

- $\tau \sim 1$; ... but the interesting region is $\tau < 0.3$
- $L_{ij} \rightarrow 0$ for $z \rightarrow \tau$; **Dynamical Threshold Enhancement**

$$\frac{d\sigma}{dM} = \frac{8\pi\beta}{3M} \int_{\tau}^1 \frac{dz}{z} \sum_{ij=(q\bar{q}, gg, \bar{q}q)} L_{ij} \left(\frac{\tau}{z}, \mu \right) C_{ij} (z, \dots, \mu)$$

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HARD SCATTERING KERNELS C_{ij}

In the limit $z \rightarrow 1$ we can distinguish three different scales

$$s, M^2, m_t^2 \gg s(1-z)^2 \gg \Lambda_{\text{QCD}}^2$$

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In the threshold region the hard scattering kernels factor into hard functions and soft functions (matrices in color space)

Kidonakis, Sterman ('97)

$$C(z, M, m_t, \mu) = \int_{-1}^1 d \cos \theta \text{Tr} \left[\mathbf{H}(M, m_t, \cos \theta, \mu) \mathbf{S}(\sqrt{s}(1-z), m_t, \cos \theta, \mu) \right]$$

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The same factorization formula can be re-obtained using the language of Soft-Collinear Effective Theory

HARD SCATTERING KERNELS C_{ij} -II

The soft functions include plus distributions of the form

$$\alpha_s^n \left[\frac{\ln^m(1-z)}{1-z} \right]_+ \quad m = 0, \dots, 2n-1$$

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In particular at NNLO

$$\begin{aligned} C_{ij}^{\text{NNLO}}(z, M, m_t, \mu) = & \int_{-1}^1 d\cos\theta \left\{ D_3 \left[\frac{\ln^3(1-z)}{1-z} \right]_+ + D_2 \left[\frac{\ln^2(1-z)}{1-z} \right]_+ \right. \\ & + D_1 \left[\frac{\ln(1-z)}{1-z} \right]_+ + D_0 \left[\frac{1}{1-z} \right]_+ \\ & \left. + C_0 \delta(1-z) + R(z) \right\} \end{aligned}$$

D_i, C_0 are functions of $M, m_t, \cos\theta, \mu$;

(D_3, D_2, D_1 first obtained by Kidonakis, Leanen, Moch, and Vogt ('01))

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In particular

$$C_{ij}^{\text{NNLO}}(z, M)$$

\mathbf{H} and \mathbf{S} obey RGE of the form

$$\frac{d}{d \ln \mu} \mathbf{H} = \boldsymbol{\Gamma}_H \mathbf{H} + \mathbf{H} \boldsymbol{\Gamma}_H^\dagger$$

where $\boldsymbol{\Gamma}$ is known up to NNLO

(AF, Neubert, Pecjak, and Yang ('09))

$$+ C_0 \delta(1-z) + R(z) \}$$

$$\left[\frac{\ln^2(1-z)}{1-z} \right]_+$$

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HARD SCATTERING KERNELS C_{ij} -II

By exploiting the information encoded in Γ and RGEs
it was possible to calculate D_3, D_2, D_1, D_0 and
the scale dependence of C_0

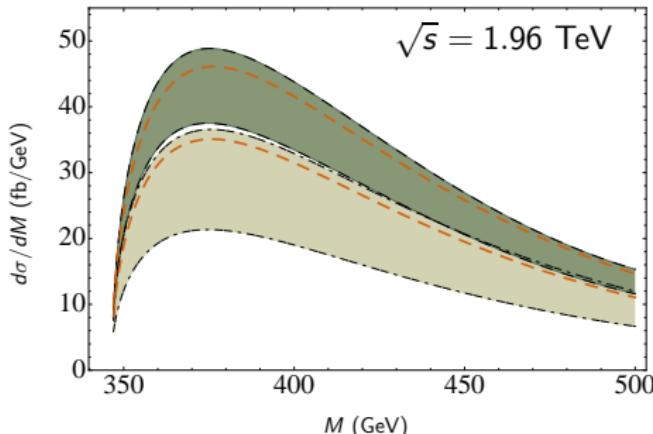
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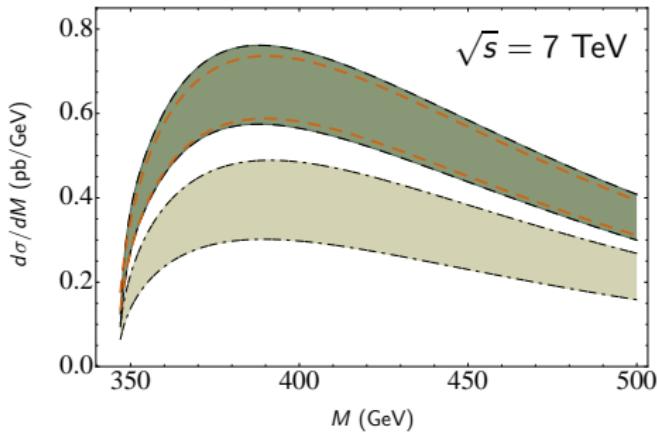
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THRESHOLD EXPANSION VS EXACT NLO

Tevatron



LHC

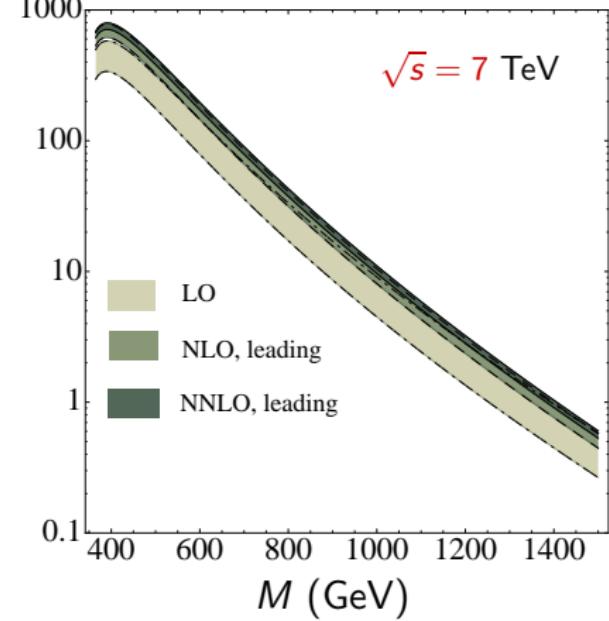
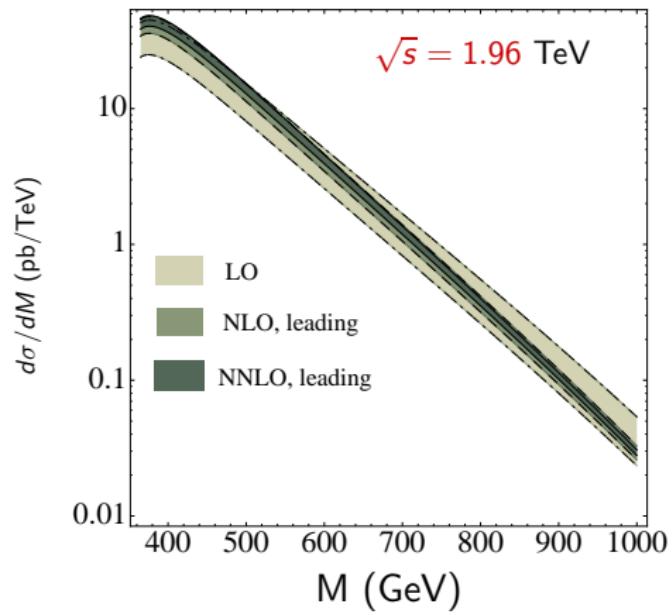


- ▶ Exact NLO result (dark grey band) obtained with MCFM (Campbell, Ellis)
- ▶ The NLO threshold expansion → band between the dashed lines ($200 \text{ GeV} \leq \mu \leq 800 \text{ GeV}$; close to $M/2 \leq \mu \leq 2M$)
- ▶ The threshold expansion agrees quite well with the exact result, even in the low invariant mass region

THRESHOLD EXPANSION AT NNLO

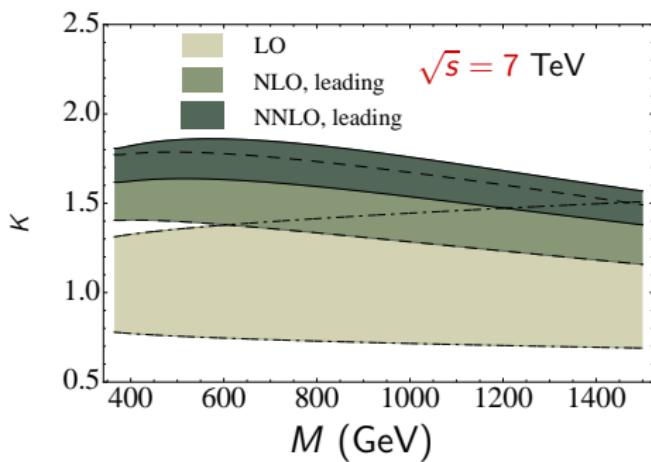
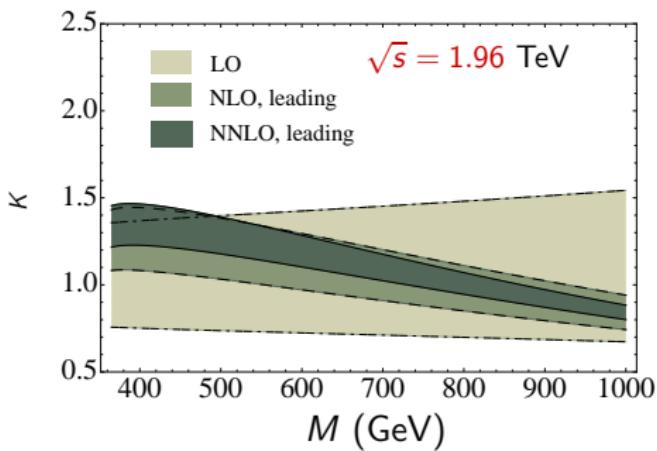
Invariant Mass Distribution

$$\left(\frac{M}{2} \leq \mu \leq 2M \right)$$



THRESHOLD EXPANSION AT NNLO

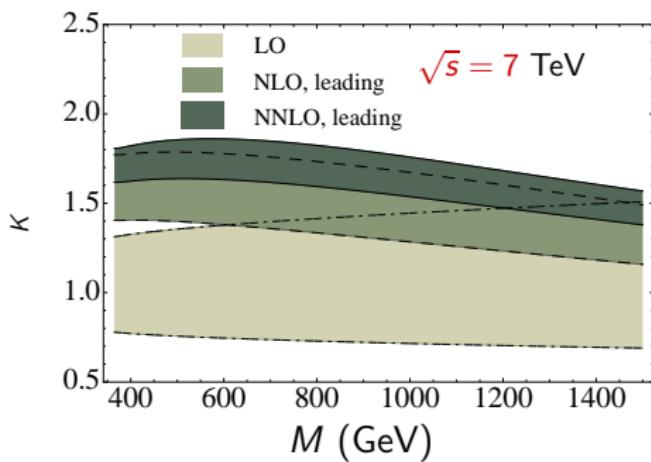
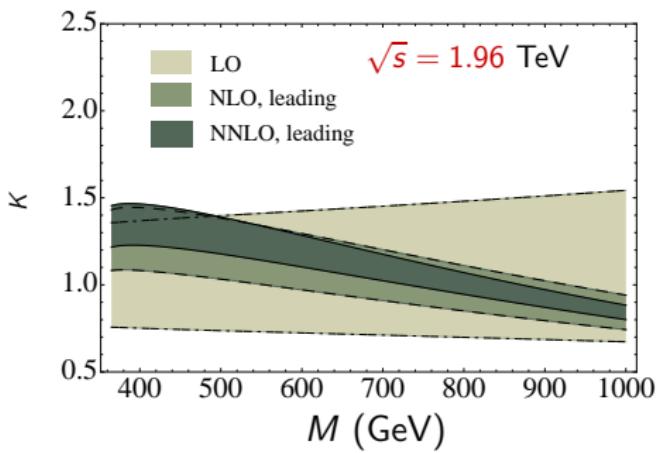
$$K = \frac{d\sigma(\mu)/dM}{d\sigma^{\text{LO}}(\mu = M)/dM}$$



Using MSTW2008 pdfs

THRESHOLD EXPANSION AT NNLO

$$K = \frac{d\sigma(\mu)/dM}{d\sigma^{\text{LO}}(\mu = M)/dM}$$



- ▶ Scale dependence still sizable
- ▶ Resummation and/or complete NNLO needed!

RESUMMATION

By solving the RGE satisfied by \mathbf{H} and \mathbf{S} it is possible to rewrite the scattering kernel as

$$C(z, M, m_t, \cos \theta, \mu_f) = \exp [4a_{\gamma^\phi}(\mu_s, \mu_f)] \\ \times \text{Tr} \left[\mathbf{U}(M, m_t, \cos \theta, \mu_h, \mu_s) \mathbf{H}(M, m_t, \cos \theta, \mu_h) \mathbf{U}^\dagger(M, m_t, \cos \theta, \mu_h, \mu_s) \right. \\ \left. \times \tilde{\mathbf{s}} \left(\ln \frac{M^2}{\mu_s^2} + \partial_\eta, M, m_t, \cos \theta, \mu_s \right) \right] \frac{e^{-2\gamma_E \eta}}{\Gamma(2\eta)} \frac{z^{-\eta}}{(1-z)^{1-2\eta}}$$

The $\ln(\mu_s/\mu_h) \sim \ln(1-z)$ are exponentiated in \mathbf{U}

$$\mathbf{U}(M, m_t, \cos \theta, \mu_h, \mu) = \mathcal{P} \exp \int_{\mu_h}^{\mu} \frac{d\mu'}{\mu'} \mathbf{\Gamma}_H(M, m_t, \cos \theta, \mu')$$

RESUMMATION AND MATCHING

All the scales can be fixed (and varied) separately:

- $\mu_h \sim \mu_f \sim M$
- μ_s is chosen to minimize the corrections coming from the soft function ($\frac{M}{4} \leq \mu_s \leq \frac{M}{10}$)

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RG-impr. PT	log accuracy	Γ_{cusp}	γ^h, γ^ϕ	$\mathbf{H}, \tilde{\mathbf{s}}$
LO	NLL	2-loop	1-loop	tree-level
NLO	NNLL	3-loop	2-loop	1-loop

All the pieces for the first NNLL calculation are now available

Ahrens, AF, Neubert, Pecjak, and Yang ('10)

RESUMMATION AND MATCHING

All the scales can be fixed (and varied) separately:

- $\mu_h \sim \mu_f \sim M$
- μ_s is chosen to minimize the corrections coming from the soft function ($M - M$)

It is possible to **match** fixed order NLO and NNLL

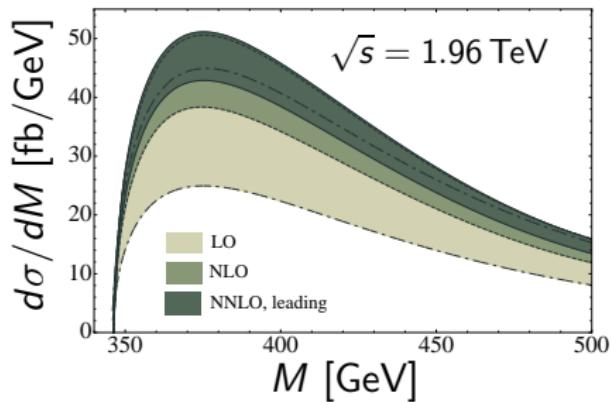
$$d\sigma^{\text{NLO+NNLL}} = d\sigma^{\text{NNLL}} \Big|_{\mu_h, \mu_s, \mu_f} + \\ + \left(d\sigma^{\text{NLO}} \Big|_{\mu_f} - d\sigma^{\text{NNLL}} \Big|_{\mu_s=\mu_h=\mu_f} \right)$$

All the pi

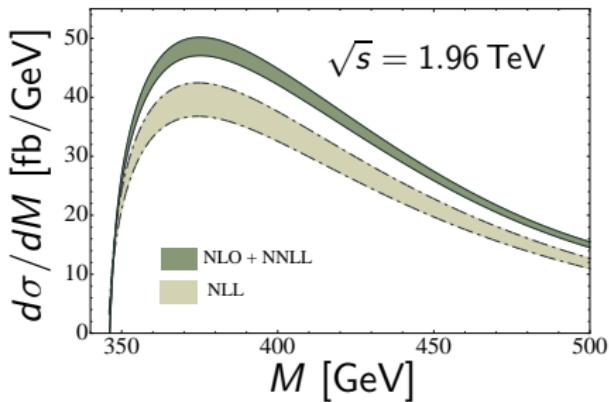
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INVARIANT MASS DISTRIBUTION AT NLO+NNLL

Tevatron



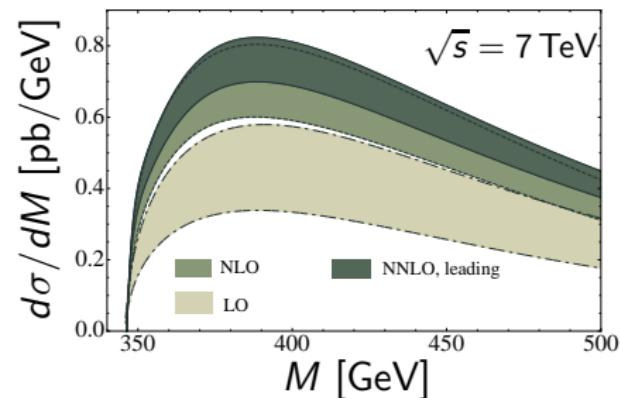
(here $\mu_f = 400$ GeV)



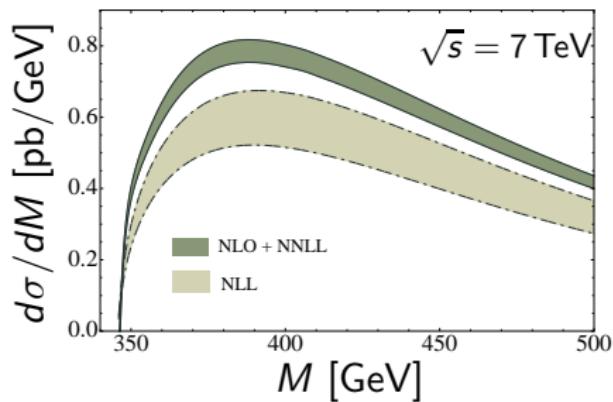
The perturbative uncertainty estimated by scale variations decreases at
NLO+NNLL

INVARIANT MASS DISTRIBUTION AT NLO+NNLL

LHC



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APPROXIMATE NNLO CALCULATIONS:

The Total Cross Section

THE TOTAL CROSS SECTION

The total cross section is obtained by integrating the distribution over M

$$\sigma(s_{\text{had}}, m_t) = \int_{2m_t}^{\sqrt{s_{\text{had}}}} dM \frac{d\sigma}{dM}$$

$\mu_f = 173 \text{ GeV}$	Tevatron [pb]	LHC (7 TeV) [pb]
$\sigma_{\text{NLO,lead}}$	$6.20^{+0.39 +0.31}_{-0.71 -0.23}$	$144^{+5 +7}_{-13 -8}$
σ_{NLO}	$6.49^{+0.33 +0.33}_{-0.70 -0.24}$	$150^{+18 +8}_{-19 -9}$
$\sigma_{\text{NNLL+NLO}}$	$6.48^{+0.17 +0.32}_{-0.21 -0.25}$	$146^{+7 +8}_{-7 -8}$

- The NLO total cross section is known analytically

Czakon and Mitov ('08)

- First error from scale variations, second from PDFs
(MSTW2008NNLO at 90%)

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$\mu_f = 400 \text{ GeV}$	Tevatron [pb]	LHC (7 TeV) [pb]
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σ_{NLO}	$5.64^{+0.73 +0.30}_{-0.75 -0.22}$	$126^{+19 +7}_{-18 -7}$
$\sigma_{\text{NNLL+NLO}}$	$6.30^{+0.19 +0.31}_{-0.19 -0.23}$	$149^{+7 +8}_{-7 -8}$

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APPROXIMATE TOTAL PARTONIC CS AT NNLO

Approximate NNLO formulas for the partonic cross section for

$$\beta = \sqrt{1 - 4m_t^2/s} \rightarrow 0$$

(production threshold region) were recently obtained

S. Moch and P. Uwer ('08)

U. Langenfeld, S. Moch, and P. Uwer ('09)

M. Beneke, M. Czakon, P. Falgari, A. Mitov, and C. Schwinn ('09)

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The approximate NNLO formulas include:

- ▶ All terms proportional to $1/\beta$ and/or $\ln \beta$
- ▶ All the **scale-dependent** terms

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M. Beneke, M. Czakon, P. Falgari, A. Mitov, and C. Schwinn ('09)

For $\mu = m_t$ in the channel $q\bar{q} \rightarrow t\bar{t}$ one finds

$$\hat{\sigma}_{q\bar{q}}^{\text{NNLO}} \propto \hat{\sigma}_{q\bar{q}}^{\text{LO}} \left\{ \frac{3.60774}{\beta^2} + \frac{1}{\beta} (-140.368 \ln^2 \beta + 32.106 \ln \beta + 3.95105) \right.$$
$$+ 910.222 \ln^4 \beta - 1315.53 \ln^3 \beta + 592.292 \ln^2 \beta$$
$$\left. + 528.557 \ln \beta + C_{q\bar{q}}^{\text{NNLO}} + \mathcal{O}(\beta) \right\}$$

APPROXIMATE TOTAL PARTONIC CS AT NNLO

Approximate NNLO formulas for the partonic cross section for

$$\beta = \sqrt{1 - 4m_t^2/s} \rightarrow 0$$

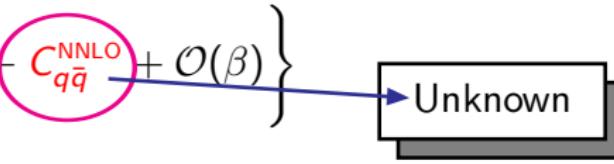
(production threshold region) were recently obtained

S. Moch and P. Uwer ('08)

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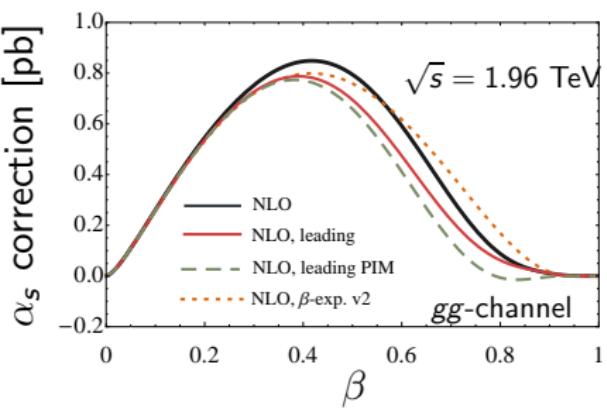
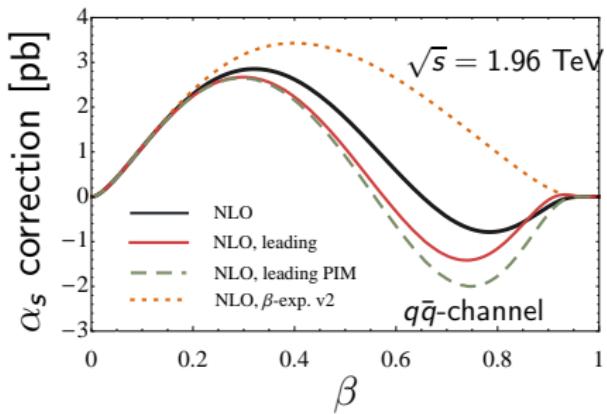
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$\hat{\sigma}_{q\bar{q}}^{\text{LO}} = \frac{\pi \beta \rho}{27} (2 + \rho) \sim \beta$
 $\left(\rho = \frac{4m_t^2}{s} \right)$

$\beta \rightarrow 0$ VS $z \rightarrow 1$ AT NLO

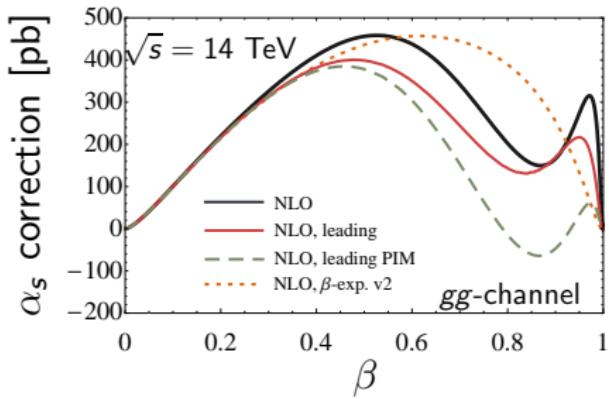
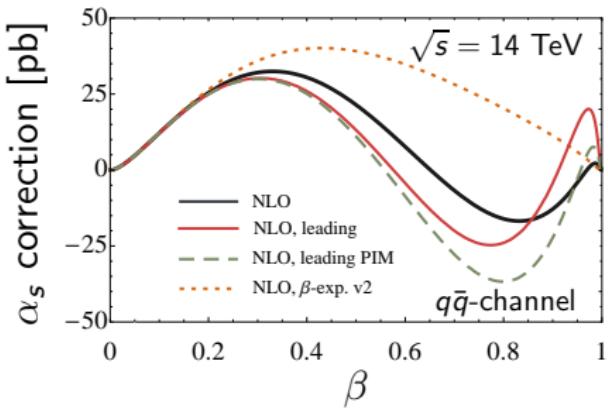
Tevatron: the total CS obtained from the $\beta \rightarrow 0$ expansion overestimates the exact result



Partonic cross sections \times luminosity

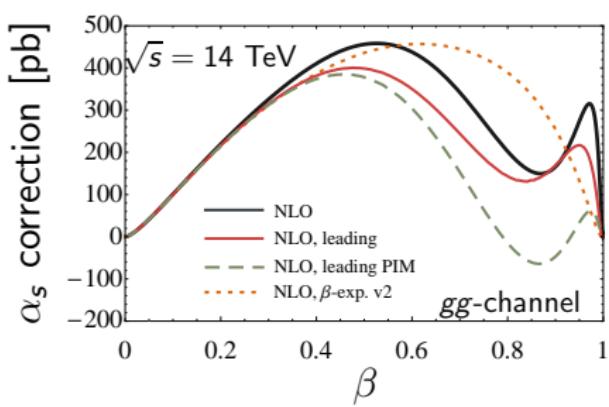
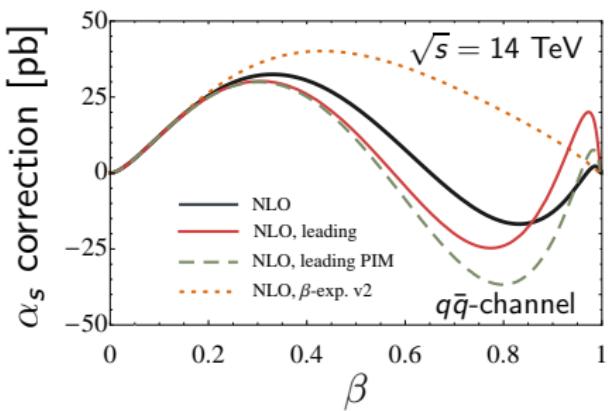
$\beta \rightarrow 0$ VS $z \rightarrow 1$ AT NLO

LHC: the small β expansion gives results which are closer to the exact result



$\beta \rightarrow 0$ VS $z \rightarrow 1$ AT NLO

LHC: the small β expansion gives results which are closer to the exact result



The leading terms in the $z \rightarrow 1$ expansion reproduce the correct shape of the exact NLO in all cases

SUMMARY & CONCLUSIONS

- The measurements related to the production of top-quark pairs play a crucial role at the Tevatron and at the LHC: **desirable to go beyond the current NLO theoretical predictions**
- Approximate NNLO and resummed NNLL results in the partonic threshold limit ($z \rightarrow 1$) are available for the **invariant mass distribution, total cross section, charge asymmetry**.
- Interesting comparisons with other approaches, possible extensions to 1PI kinematics

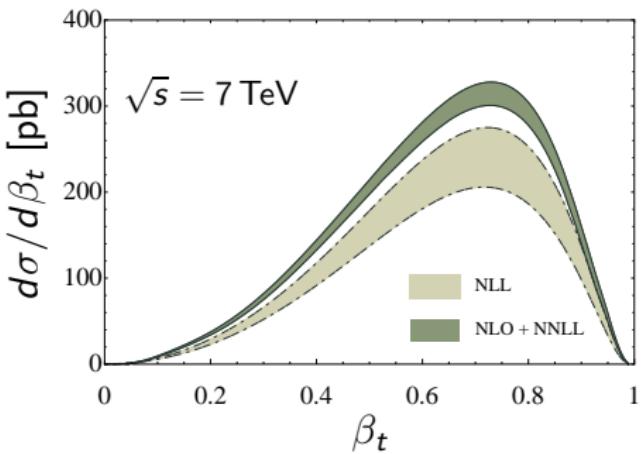
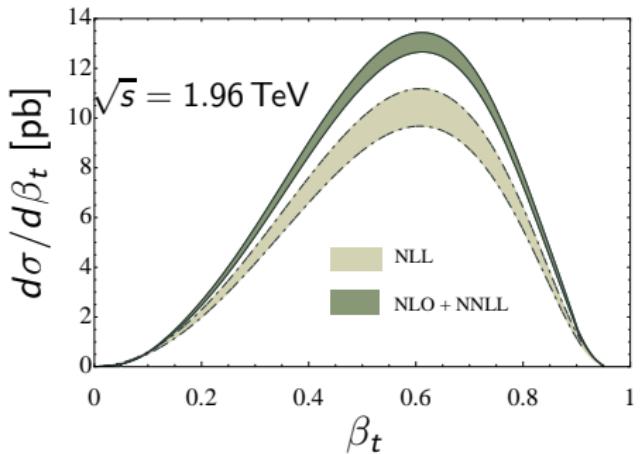
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Complete NNLO calculations are still welcome!
(and in progress, but this is another story...)

Backup Slides

DISTRIBUTIONS IN β_t

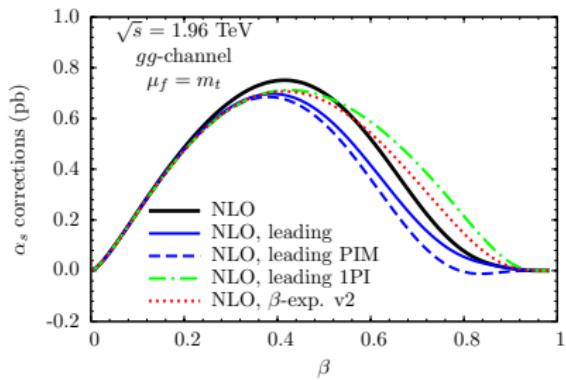
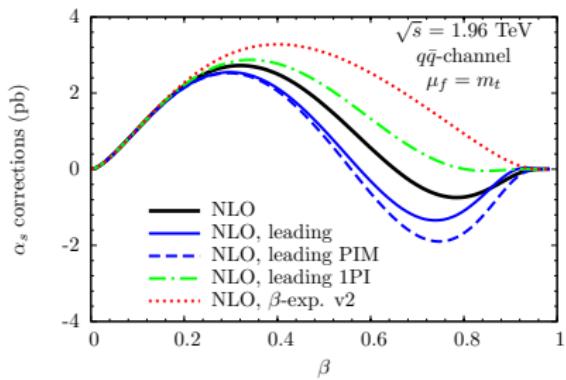


Distributions $d\sigma/d\beta_t$ at the Tevatron (left) and LHC (right)

$$\beta_t = \sqrt{1 - \frac{4m_t^2}{M^2}}$$

1PI KINEMATICS

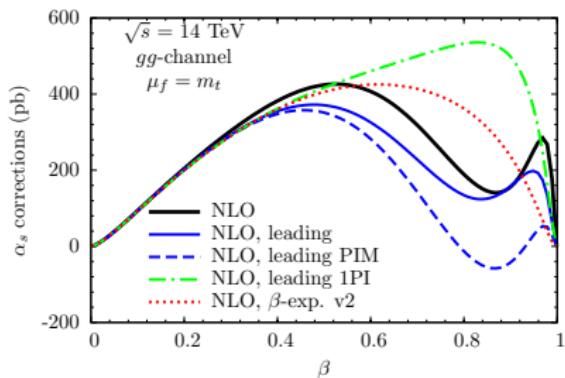
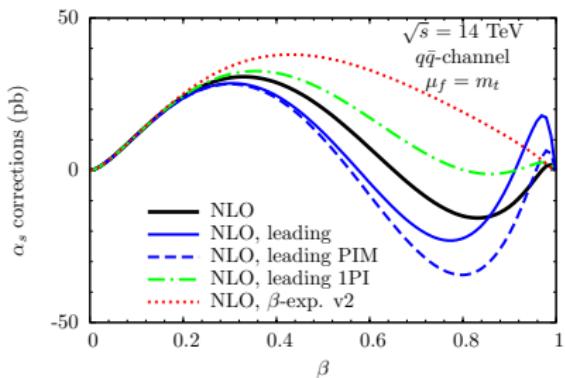
Tevatron



Preliminary

1PI KINEMATICS

LHC



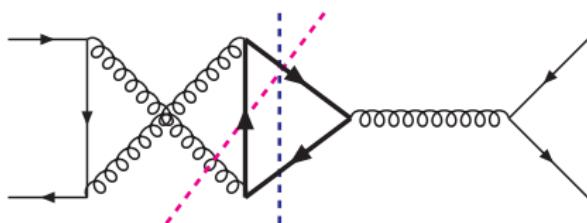
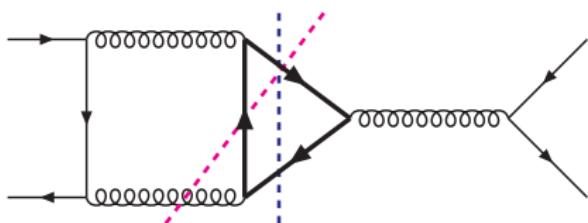
Preliminary

CHARGE ASYMMETRY

The charge asymmetry is the difference in production rate for top and antitop at fixed angle or rapidity

$$\underbrace{A(y) = \frac{N_t(y) - N_{\bar{t}}(y)}{N_t(y) + N_{\bar{t}}(y)}}_{\text{differential CA}} \quad \underbrace{A = \frac{N_t(y \geq 0) - N_{\bar{t}}(y \geq 0)}{N_t(y \geq 0) + N_{\bar{t}}(y \geq 0)}}_{\text{integrated CA}} \quad \left(N_i \equiv \frac{d\sigma^{t\bar{t}}}{dy_i} \right)$$

Arising at order α_s^3 in the channel $q\bar{q} \rightarrow t\bar{t}$



THE FORWARD-BACKWARD ASYMMETRY AT TEVATRON

Because of QCD charge conjugation invariance, $N_t(y) = N_t(-y)$, and therefore A is equal to the **forward-backward asymmetry**

$$A_{\text{FB}} \equiv \frac{1}{\sigma} \int_{2m_t}^{\sqrt{s}} dM \left(\int_0^1 d \cos \theta \frac{d^2 \sigma^{N_1 N_2 \rightarrow t\bar{t}X}}{dM d \cos \theta} - \int_{-1}^0 d \cos \theta \frac{d^2 \sigma^{N_1 N_2 \rightarrow t\bar{t}X}}{dM d \cos \theta} \right)$$

- The measured asymmetry in the lab frame

$$A_{\text{FB}}^{p\bar{p}} = 19.3 \pm 6.9\%$$

- The predicted LO asymmetry is $A_{\text{FB}}^{p\bar{p}} = 5.1_{-0.3}^{+0.7}\%$

Kühn and Rodrigo ('08), Bernreuther and Si ('10)

- In the $t\bar{t}$ -frame the asymmetry is $\sim 30\%$ larger

$$A_{\text{FB}}^{t\bar{t}} = 24 \pm 13\%$$

THE FORWARD-BACKWARD ASYMMETRY AT TEVATRON

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$$A_{FB} \equiv \frac{\int_0^{\sqrt{s}} \int^1 d^2\sigma N_1 N_2 \rightarrow t\bar{t}X}{\int_0^0 d^2\sigma N_1 N_2 \rightarrow t\bar{t}X} \Big|_{\theta}$$

• Theory	AFNPY ('10) partonic frame	$A_{FB}(\%)$ ($\mu_f = m_t$)	$A_{FB}(\%)$ ($\mu_f = 400$ GeV)
• Theory	"LO" QCD	$7.3^{+0.7}_{-0.6}$	$6.6^{+0.5}_{-0.5}$
• Theory	"LO" + NNLL	$7.3^{+1.1}_{-0.7}$	$6.6^{+0.6}_{-0.5}$

$A_{FB}^{t\bar{t}} = 7.6^{+0.8}_{-0.5}$ LO QCD ; $A_{FB}^{t\bar{t}} = 8.0^{+0.7}_{-0.5}$ LO QCD + EW
Bernreuther and Si ('10)

• In

$$A_{FB}^{t\bar{t}} = 24 \pm 13\%$$

('10)

IR POLES IN QCD AMPLITUDES

IR poles in QCD amplitudes can be removed by a multiplicative renormalization

Becher and Neubert ('09)

$$Z^{-1}(\epsilon, \{p\}, \{m\}) |\mathcal{M}_n(\epsilon, \{p\}, \{m\})\rangle_{\alpha_s^{QCD} \rightarrow \xi \alpha_s} = \text{FINITE}$$

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$$\mathbf{Z}^{-1} = \mathbf{1} - \alpha_s \mathbf{Z}_{(1)} + \alpha_s^2 (\mathbf{Z}_{(1)}^2 - \mathbf{Z}_{(2)}) + \mathcal{O}(\alpha_s^3)$$

$$\mathcal{M} = \alpha_s \mathcal{M}^{(0)} + \alpha_s^2 \mathcal{M}^{(1)} + \alpha_s^3 \mathcal{M}^{(2)} + \mathcal{O}(\alpha_s^4)$$

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therefore

$$|\mathcal{M}_n^{(1), \text{sing}}\rangle = \mathbf{Z}^{(1)} |\mathcal{M}_n^{(0)}\rangle$$

$$|\mathcal{M}_n^{(2), \text{sing}}\rangle = \left[\mathbf{Z}^{(2)} - \left(\mathbf{Z}^{(1)} \right)^2 \right] |\mathcal{M}_n^{(0)}\rangle + \left(\mathbf{Z}^{(1)} |\mathcal{M}_n^{(1)}\rangle \right)_{\text{poles}}$$

But what is \mathbf{Z} ?

EVOLUTION MATRIX

\mathbf{Z} satisfies the evolution equation

$$\mathbf{Z}^{-1}(\epsilon, \{\underline{p}\}, \{\underline{m}\}, \mu) \frac{d}{d \ln \mu} \mathbf{Z}(\epsilon, \{\underline{p}\}, \{\underline{m}\}, \mu) = -\mathbf{\Gamma}(\{\underline{p}\}, \{\underline{m}\}, \mu)$$

where, in the color space formalism

$$\begin{aligned} \mathbf{\Gamma} &= \sum_{(i,j)} \frac{\mathbf{T}_i \cdot \mathbf{T}_j}{2} \gamma_{\text{cusp}}(\alpha_s) \ln \frac{\mu^2}{-s_{ij}} + \sum_i \gamma^i(\alpha_s) \\ &- \sum_{(I,J)} \frac{\mathbf{T}_I \cdot \mathbf{T}_J}{2} \gamma_{\text{cusp}}(\beta_{IJ}, \alpha_s) + \sum_I \gamma^I(\alpha_s) + \sum_{I,j} \mathbf{T}_I \cdot \mathbf{T}_j \gamma_{\text{cusp}}(\alpha_s) \ln \frac{m_I \mu}{-s_{Ij}} \\ &+ \sum_{(I,J,K)} i f^{abc} \mathbf{T}_I^a \mathbf{T}_J^b \mathbf{T}_K^c F_1(\beta_{IJ}, \beta_{JK}, \beta_{KI}) \\ &+ \sum_{(I,J)} \sum_k i f^{abc} \mathbf{T}_I^a \mathbf{T}_J^b \mathbf{T}_k^c f_2\left(\beta_{IJ}, \ln \frac{-\sigma_{Jk} v_J \cdot p_k}{-\sigma_{Ik} v_I \cdot p_k}\right) + \mathcal{O}(\alpha_s^3) \end{aligned}$$

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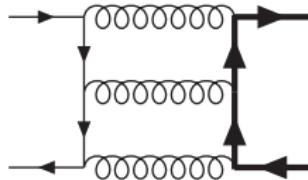
Staring at two-loop order one finds
three particle correlators

$$\begin{aligned} \mathbf{\Gamma} &= \sum_{(i,j)} \text{The explicit expression for the coefficient} \\ &\quad \text{functions } F_1 \text{ and } f_2 \text{ was recently obtained} \\ &- \sum_{(I,J)} \text{AF, Neubert, Pecjak, Yang ('09)} \\ &+ \sum_{(I,J,K)} i f^{abc} \mathbf{T}_I^a \mathbf{T}_J^b \mathbf{T}_K^c F_1(\beta_{IJ}, \beta_{JK}, \beta_{KI}) \\ &+ \sum_{(I,J)} \sum_k i f^{abc} \mathbf{T}_I^a \mathbf{T}_J^b \mathbf{T}_k^c f_2\left(\beta_{IJ}, \ln \frac{-\sigma_{Jk} v_J \cdot p_k}{-\sigma_{Ik} v_I \cdot p_k}\right) + \mathcal{O}(\alpha_s^3) \end{aligned}$$

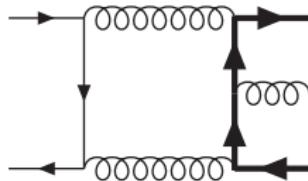
Becher and Neubert ('09)

NNLO LAUNDRY LIST

- Two-loop diagrams with a $t\bar{t}$ in the final state



- One-loop diagrams with a $t\bar{t}g(q, \bar{q})$ in the final state

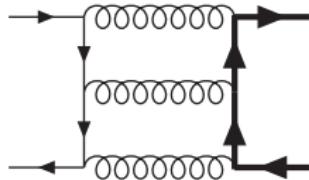


- Tree-level diagrams with a $t\bar{t}gg(gq, g\bar{q}, q\bar{q})$ in the final state



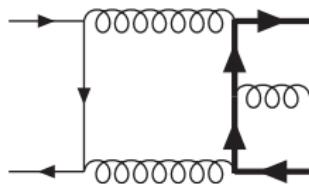
NNLO LAUNDRY LIST

- Two-loop diagrams with a $t\bar{t}$ in the final state



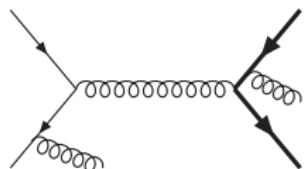
- ▶ 200 diagrams in $q\bar{q} \rightarrow t\bar{t}$
- ▶ 800 diagrams in $gg \rightarrow t\bar{t}$

- One-loop diagrams with a $t\bar{t}g(q, \bar{q})$ in the final state



Dittmaier, Uwer, Wenzierl ('07, '08)
Bevilacqua *et al.* ('10)
Melnikov, Schulze ('10)

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Two-Loop Corrections to $q\bar{q} \rightarrow t\bar{t}$

The two-loop corrections to $q\bar{q} \rightarrow t\bar{t}$ were first evaluated in the **limit** in which $s, |t|, |u| \gg m_t^2$

M. Czakon, A. Mitov, and S. Moch ('07)

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Adding more terms in the expansion in powers of $m^2/s, m^2/|t|, m^2/|u|$ it is not sufficient for phenomenological studies (particularly near threshold)

An **exact numerical** evaluation of these correction is available

M. Czakon ('08)

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Analytic calculations:

- Diagrams with a closed (light or heavy) quark loop

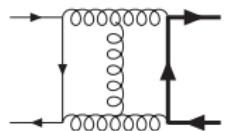
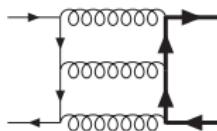
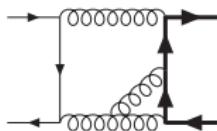
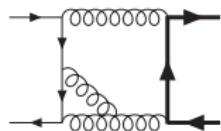
Bonciani, AF, Gehrmann, Maître, Studerus ('08)

- Leading color coefficient in the N_c expansion (planar diagrams only)

Bonciani, AF, Gehrmann, Studerus ('09)

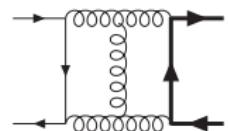
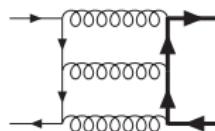
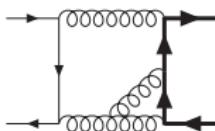
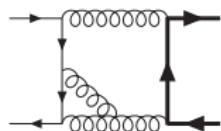
SOME TECHNICAL DETAILS

The leading color coefficient involves planar diagrams only



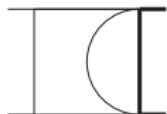
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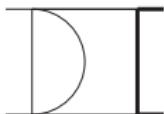


A new GiNaC/C++ implementation of the Laporta Algorithm was developed for this project: **REDUZE** ([Studerus \('09\)](#))

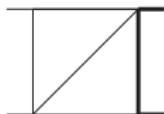
It involves 6 new irreducible box topologies



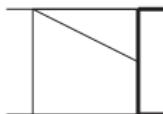
2 MIs



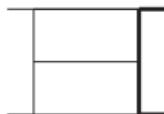
2 MIs



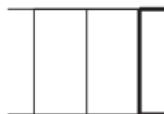
2 MIs



2 MIs



2 MIs



3 MIs

C. Studerus ('09)

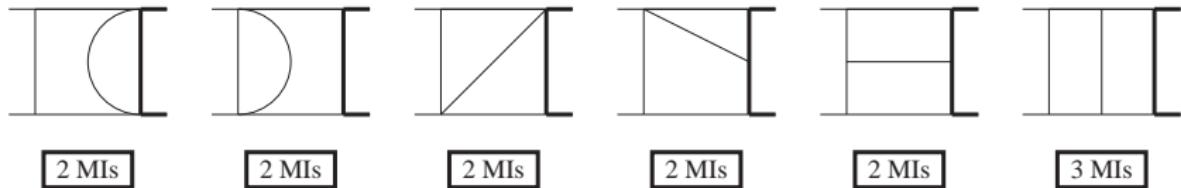
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[C. Studerus \('09\)](#)

The use new **two-dim HPLs** is unavoidable to obtain analytic expressions; they can be expanded analytically and evaluated numerically with a GiNaC package ([Vollinga and Weinzierl \('04\)](#))

Two-Loop Corrections to $gg \rightarrow t\bar{t}$

- The two-loop diagrams in the $gg \rightarrow t\bar{t}$ channel are available only in the $s \gg m_t^2$ limit

Czakon, Mitov, and Moch ('08)

- The coefficients of all the IR poles are known analytically (see talk by L. Yang)

AF, Neubert, Pecjak, and Yang ('09)

- The diagrams involving massless quark loops can be calculated analytically in the usual way

Bonciani, AF, Gehrmann, Studerus (in progress)

- Part of the virtual corrections involve many MI which cannot be expressed in terms of HPLs only

$$\downarrow p^2 \neq -m^2 \quad \Rightarrow \quad \text{Elliptic Functions} \\ K(z) = \int_0^1 \frac{dx}{\sqrt{(1-x^2)(1-zx^2)}} \\ \text{Laporta Remiddi ('04)}$$
